Covariant Perturbation Theory of Non-Abelian Kinetic Theory

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Abstract

A new 'double perturbation' theory is presented in the framework of the kinetic theory of quark-gluon plasma. A solvable set of equations from the 'double perturbation' is derived and are shown to be gauge-invariant. As an example, the Landau damping rate for the plasmon at zero momentum is calculated and discussed.

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In the last decade, much interest was focused on the study of non-Abelian plasma from transport theory [1]. It is generally believed that kinetic theory can describe correctly the quark-gluon plasma (QGP) physics just as what can be done by the temperature field theory [3], and is ready to be extended to out-of-equilibrium situations [4]. It has been demonstrated that the hard thermal loop (HTL) in temperature field theory can be obtained from the QGP kinetic theory [2, 5], but up to now there does not exist a valid scheme for solving the kinetic equations and at the same time keeping the non-Abelian gauge symmetry.

A standard perturbative approach to the kinetic equations of plasma, which can keep the guage symmetry, is to expand the equations in the ascending power of guage coupling constant [2]. Another popular method in traditional study of plasma is to expand the equations in powers of weak field strength [4, 6, 7]. Since these two methods are in full

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agreement with each other in the study of electromagnetic plasma [8, 9] due to the Abelian nature of the dynamics (linear dynamics), they have been widely used in the literatures for QGP. However, they both suffer from powerlessly overcome shortcomings: The former, as is well known, gives only a linear leading order, i.e. an Abelian-like contribution, instead of truly non-Abelian ones, while the latter breaks the non-Abelian gauge covariance badly as was pointed out in Ref. [2, 7].

If we want to solve the above-mentioned problem in the framework of kinetic theory, we must pay special attention to the following two aspects: the preserve of gauge invariance in the perturbation process and the treatment of the nonlinearity in the equations because of the nonlinear (non-Abelian) nature of QGP.

In the present letter, we will propose a new scheme both satisfying the SU(3) gauge symmetry and having solvability for non-Abelian kinetic theory. The basic idea of the scheme is: After expanding the kinetic equations in guage coupling constant g, do the iterative calculation in powers of field strength for the purely non-Abelian counterpart. In the following we will call this scheme as 'double perturbation'.

Our aim is to provide a well-defined prescription for treating the color dielectric physics. In this letter, we will first present the framework of the new perturbation theory and then give the main results of our analysis. This includes the derivation of a general set of perturbative kinetic equations for non-Abelian plasma and the comments of the gauge invariance and solvability of these equations. An application of the new formalism in the explicit calculation of Landau damping rate in close-to-equilibrium QGP, which has been computed from HTL in field theory [10, 11], will be given as an example. Finally, we will show some evidence that our results are associated with or even go beyond the HTL approximation. A more extensive discussion and further details on computations will be given in a longer article [12].

Let us begin with the derivation of a set of equations describing the dynamics associated with non-Abelian fluctuations. It is sufficient to adopt the semiclassical kinetic theory of QGP for studying the thermal effects. The theory consists of kinetic equations and field equation. The kinetic equations read as [13]

$$p^\mu D_\mu Q_\pm(\mathbf{p}, x) \pm \frac{g}{2} p^\mu \partial_p^\nu \{F_{\mu\nu}(x), Q_\pm(\mathbf{p}, x)\} = 0, \tag{1}$$

$$p^\mu \tilde{D}_\mu G(\mathbf{p}, x) + \frac{g}{2} p^\mu \partial_p^\nu \{\tilde{F}_{\mu\nu}(x), G(\mathbf{p}, x)\} = 0, \tag{2}$$

where the letters with ˜ represent the corresponding operators in adjoint representation of SU(N), in which the generator is T_a.

The background field equation is written as

$$D^{\mu}F^a_{\mu\nu} = j^a_{\nu}(x), \tag{3}$$

where covariant derivative $D^{\mu} = \partial^{\mu} + igA^{\mu}(x)$, field potential $A^{\mu} \equiv A^{\mu}_a \tau^a$, and field tensor $F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu} - igf_{abc}\tau^c A^{\mu}_a A^{\nu}_b$. τ_a is the generator of SU(N). The color currents including external and induced ones are $j^a_{\nu} = j^{\text{ext } a}_{\nu} + j^{\text{ind } a}_{\nu}$.

In principle, a perturbative method is often necessary to solve nonlinear equations. Here, for simplicity, we take the close-to-equilibrium situation of a QGP as an example to discuss this method. We will show later that our method is easily extended to out-of-equilibrium QGP.

Following the same philosophy in traditional kinetic theory we write the distribution functions $Q_{\pm}(x,p), G(x,p)$ and fields as [2, 13],

$$A^a_{\mu} = a^a_{\mu}, \quad Q_{\pm} = Q^{\text{eq}}_{\pm} + \delta Q_{\pm}, \quad G = G^{\text{eq}} + \delta G. \tag{4}$$

The associated density fluctuations and induced fields are random quantities, satisfying $\langle \delta Q \rangle = 0, \langle \delta G \rangle = 0$ and $\langle a \rangle = 0$, where symbol $\langle \ \rangle$ denotes the random-phase average. We define the field tensor corresponding to a_{μ} as $f_{\mu\nu}$.

In this way we obtain a set of kinetic equations for fluctuations from equations (1) and (2)

$$p^{\mu}D_{\mu}\delta Q_{\pm}(\mathbf{p},x) \pm \frac{g}{2}p^{\mu}\partial^{\nu}_p\{f_{\mu\nu}(x), Q_{\pm}(\mathbf{p},x)\} = 0, \tag{5}$$

$$p^{\mu}\tilde{D}_{\mu}\delta G(\mathbf{p},x) + \frac{g}{2}p^{\mu}\partial^{\nu}_p\{\tilde{f}_{\mu\nu}(x), G(\mathbf{p},x)\} = 0. \tag{6}$$

In order to ensure the consistency of soft covariant derivative, in the following we will impose a limitation on the amplitude of the fields: if $a_{\mu} \sim T$, then $ga_{\mu} \sim gT$ is of the same order as the derivative of a slowly varying quantity, $i\partial_{\mu} \sim ga_{\mu}$ [14].

We now employ our 'double perturbation' scheme to expand the distribution functions in powers of g in the first step, and then iterate repeatedly the nonlinear parts in the expanded equations in field quantity. These can be expressed as

$$\delta Q = gQ^{(1)} + g^2Q^{(2)} + \cdots, \quad \delta G = gG^{(1)} + g^2G^{(2)} + \cdots, \tag{7}$$

$$Q^{(n)} = \sum_{\lambda=1} Q^{(n,\lambda)}, \quad G^{(n)} = \sum_{\lambda=1} G^{(n,\lambda)}. \tag{8}$$

Thus we obtain a series of equations

$$p^{\mu}\partial_{\mu}Q^{(n)}_{\pm} \pm ig\sum_{\lambda} p^{\mu}[a_{\mu}, Q^{(n,\lambda)}_{\pm}] + \frac{g}{2}p^{\mu}\{f_{\mu\nu}, \partial^{\nu}_p Q^{(n-1)}_{\pm}\} = 0, \tag{9}$$

$$p^\mu \partial_\mu G^{(n)} + ig\sum_\lambda p^\mu[\tilde{a}_\mu, G^{(n,\lambda)}] + \frac{g}{2}p^\mu\{\tilde{f}_{\mu\nu}, \partial_p^\nu G^{(n-1)}\} = 0, \tag{10}$$

$$D^\mu(a) f_{\mu\nu}^{(n)a} = j_\nu^{(n)\text{ind }a}(x), \tag{11}$$

where n and λ represent the powers of coupling constant and field quantity, respectively. The induced currents associated with each order of density fluctuations are determined by

$$j_\nu^{(n)\text{ind }a} = g\int \frac{d^3p}{(2\pi)^3}\frac{p_\nu}{p_0}\text{Tr}\left(2N_f\tau^a[Q_+^{(n)} - Q_-^{(n)}] + 2T^a G^{(n)}\right) \tag{12}$$

where N_f is the number of quark flavour. The factor 2 accounts for the spin degrees of freedom.

Equations (9), (10), (11) and (12) form a basis of perturbation theory of non-Abelian kinetic theory.

Some short comments follow.

1. The non-Abelian gauge symmetry is exactly preserved in the perturbation equations (9) and (10) for each order of g. As a consequence $Q_\pm^{(n)}$ and $G^{(n)}$, like $Q_\pm$ and G, transform separately as SU(3) gauge-invariant scalars. Because the summation of non-Abelian terms in (9) and (10) can be done infinitely, the 'double perturbation' method we use guarantees the gauge-invariance of the result of a physical quantity within any high precision (λ being an arbitrarily large number).

2. In general, it is a difficult task to straightforwardly solve the equations (5) and (6) due to the the nonlinearity of non-Abelian counterparts [15]. However, the summation over λ in equations (9) and (10) imply that the iterative procedure in the powers of field quantity ensures the solvability of the non-Abelian counterparts.

3. As was pointed out in Ref. [16], the perturbative expansion in field quantity only, which had been carried out by some authors [4, 6, 7], have to suffer from the disadvantage of breaking non-Abelian gauge symmetry at each step of approximation calculation. This is because the results of all higher orders in field quantity contain the contributions from the relevant lower orders in coupling constant g for the density fluctuations. So, in order to get a gauge-invariant physical result, the problem of resummation of all the contributions of the same order in g has to be taken into account in the past works [6, 7, 16]. While our approach automatically singles out all the contributions of the same order in g and collects them together. In this sense, the treatment of non-Abelian contributions in our approach gives a theoretical basis for the resummation technique.

4. It can be easily verified from Eq.(12) that not only the total color current but also

each order current obey the covariant conservation law

$$D^\mu j_\mu^{(n)\mathrm{ind}\ a} = 0. \tag{13}$$

This is automatically consistent with field equation (11).

5. Our theory can be easily generalized to the out-of-equilibrium situations through the decomposion of the distribution functions $Q_\pm(x,p), G(x,p)$ and fields A_μ into regular terms and density fluctuations. The expression (4) is then replaced by

$$A_\mu^a = \langle A_\mu^a \rangle + a_\mu^a, \quad Q_\pm = \langle Q_\pm \rangle + \delta Q_\pm, \quad G = \langle G \rangle + \delta G. \tag{14}$$

Such kind of division has been used in the study of QGP [4] as well as in other works [7, 15, 16]. In particular, the gauge consistency of the decomposition has been proved in Ref. [15]. Our stress here is to put forward a gauge-consistent fluctuation dynamics($a_\mu \sim T$), while Litim et al think that the regular parts describe mean field dynamics [15] in full accordance with the effective soft field dynamics($\langle A_\mu \rangle \sim gT$) [17].

6. The 'double perturbation' approach gives an insight into the plasma with color freedom of degrees.

We will get a more penetrating understanding of these remarks from the calculations below.

As an example, we now apply the above formalism to calculate the gloun damping rate for a purely gluonic gas in the close-to-equilibrium situation. The first order equation in coupling constant is

$$p^\mu \partial_\mu G^{(1)} + ig \sum_\lambda p^\mu [\tilde{a}_\mu, G^{(1,\lambda)}] + g p^\mu \tilde{f}_{\mu\nu}^{(1)} \partial_p^\nu G^{(0)} = 0 \tag{15}$$

$$D^\mu(a) f_{\mu\nu}^{(1)a} = j_\nu^{(1)\mathrm{ind} a}(x), \tag{16}$$

Denoting the summation term by $\tilde{S}$, we have

$$\begin{aligned}
\tilde{S} = \ & - \int \frac{d^4k}{(2\pi)^4} \partial_p^\nu G^{(0)} \left(ig \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \delta(k-k_1-k_2) \frac{1}{p\cdot k_2} \left[p\cdot\tilde{a}(k_1), p^\mu \tilde{f}_{\mu\nu}^{(1)}(k_2)\right] \right. \\
& + ig^2 \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \frac{d^4k_3}{(2\pi)^4} \delta(k-k_1-k_2-k_3) \frac{1}{p\cdot(k_2+k_3) p\cdot k_3} \\
& \qquad \times [p\cdot\tilde{a}(k_1), [p\cdot\tilde{a}(k_2), p^\mu \tilde{f}_{\mu\nu}^{(1)}(k_3)]] \\
& \left. + \ \cdots\cdots \right)
\end{aligned} \tag{17}$$

It is worth noticing that all the terms in $\tilde{S}$ are of same order in g. This is because the factor g in the numerator and the 4-dimensional wave vector k_μ ($\sim \partial_\mu \sim g$) in the

denominator appear in pairs and cancell the factor g each other. The remaining dependency on g, coming from the field strength $\tilde{f}^{(1)}_{\mu\nu}$, is of first order.

A general formula of $G^{(1)}(\omega, \mathbf{k})$ in the momentum space can be written down from Eq.(15). We know that the current induced by the fluctuations can be expressed as

$$j_\nu^{(1)\text{ind }a} = g \int \frac{\mathrm{d}^3\mathrm{p}}{(2\pi)^3} \frac{p_\nu}{p_0} \mathrm{Tr}[2T_a G^{(1)}(\mathbf{p}, x)]\}, \tag{18}$$

Following the same method in [4], we can derive the response equation of medium from kinetic and field equations. As an example, we do this approximately by taking λ up to 2 and consider the case of zero momentum. We get from field equation (16) together with equations (15) and (18),

$$\begin{aligned}
&-\omega^2 \varepsilon^{(\sigma)}(\omega, 0)\langle a^2(\omega, 0)\rangle \\
&\quad = Ng^2 \int \frac{d\omega_1 d^3k_1}{(2\pi)^4} d\omega_2 \delta(\omega - \omega_1 - \omega_2) \frac{1}{\omega^2 \varepsilon^{(\sigma)}} \\
&\qquad\qquad \times (\kappa_{ll}\langle a_l^2(\omega_1, \mathbf{k_1})\rangle\langle a_l^2(\omega_2, \mathbf{k_1})\rangle + \kappa_{tl}\langle a_t^2(\omega_1, \mathbf{k_1})\rangle\langle a_l^2(\omega_2, \mathbf{k_1})\rangle \\
&\qquad\qquad\qquad + \kappa_{tt}\langle a_t^2(\omega_1, \mathbf{k_1})\rangle\langle a_t^2(\omega_2, \mathbf{k_1})\rangle) \\
&\quad + Ng^2 \int \frac{d^4k_1}{(2\pi)^4} (\langle \mathbf{a}^2(k_1)\rangle - \langle a^2(k_1)\rangle)\langle a^2(\omega, 0)\rangle \\
&\quad + Ng^2 m_g^2 \int \frac{d^4k_1}{(2\pi)^4} d\mathbf{v} \frac{1}{\omega} \frac{1}{v \cdot (k_1 - k)} \left(\frac{\omega_1}{v \cdot k_1} - 1\right) \langle (\mathbf{v}\cdot\mathbf{a})^2(k_1)\rangle\langle (\mathbf{v}\cdot\mathbf{a})^2(\omega, 0)\rangle = 0,
\end{aligned} \tag{19}$$

where $\varepsilon^{(\sigma)}$ denotes the dielectric function for σ mode, $\sigma = l$ or t represent the longitudinal or transverse wave, respectively. $\kappa_{ll}, \kappa_{tl}, \kappa_{tt}$ are coefficient functions of $\omega_1, \omega_2, \mathbf{k}_1$, associated with the definite interactions of plasmons.

We know that damping is connected with the imaginary parts in Eq.(19). Obviously, the second term of the right-hand side has no imaginary part. The damping will originate from the physical processes described by the first and third terms. Therefore, the Landau damping rate can be obtained easily:

$$\gamma^{(\sigma)}(0) = (a^s + a^c)\frac{Ng^2 T}{24\pi}, \tag{20}$$

with

$$a^s = 6 \int \mathbf{k}_1^2 dk_1 d\omega_1 d\omega_2 \delta(m_g - \omega_1 - \omega_2)(K_{ll}\rho_l\rho_l + K_{tl}\rho_t\rho_l + K_{tt}\rho_t\rho_t)$$

$$a^c = 24\pi \int k_1^2 dk_1 d\omega_1 (\frac{(\omega_1 - m_g)^3}{m_g k_1 \omega_1^4}\rho_l(k_1) - \frac{\omega_1 - m_g}{m_g k_1 \omega_1}(1 - \frac{(\omega_1 - m_g)^2}{k_1^2})\rho_t(k_1)).$$

where $K_{ll} = \frac{\mathbf{k_1}^4}{\omega_1^3\omega_2^3}\kappa_{ll}, K_{tl} = -\frac{\mathbf{k_1}^2}{\omega_1\omega_2^3}\kappa_{tl}, K_{tt} = \frac{1}{\omega_1\omega_2}\kappa_{tt}$, ρ_l and ρ_t represent the longitudinal and transverse spectral densities, respectively.

The forms of a^s and a^c show that two typical physical processes contribute to the Landau damping. With the constraint of the δ function, The first one describes the self-coupling

interaction of a mode, which has a good formal correspondence to the result of HTL in field theory [11]. The other one represents the collisional interaction of two plasmons or quasiparticles, which describes the long-range interaction in QGP. The numerical result of a^c has been obtained to be 5.973, while the magnitude of a^s is estimated to be of well-matched order with a^c.

In conclusion, we have proposed a 'double perturbation' approach and derived a series of perturbation equations for the non-Abelian kinetic theory. Our approach gives a new and deeper perspective to the perturbation theory and thereby provides a progress in the methodological problem in the study of non-Abelian kinetic theory.

The study of the dielectric physics in QGP will be advanced by applying this new theory. This includes to get a non-Abelian gauge-consistent dielectric tensor and discuss the response of a QGP to external sources [4, 18], and so on. We would emphatically point out that since our approach has led λ-point functions (or correlators) into first-order fluctuations in coupling constant g, the physics beyond an equilibrium state will play more vital role in a QGP than in an electromagnetic plasma [4, 7].

As an example, we have studied in the last part of the present letter the Landau damping in the zero momentum case using this approach. The result shows that both the self-coupling interaction of a mode and the interaction of two plasmons contribute to the damping rate. The physical mechanism for two-plasmon interaction can clearly be expressed as a two-body 'collision' from long-range interaction of two quasiparticles. We need to make it clear further as for the self-coupling interaction, which has showed, in form, the similar 2-2 scattering and 2-3 scattering processes contained in the self-energy of HTL [11].

We also believe that this approach will provide more and better results to uncover the deeper link between kinetic theory and field theory searched by many investigators [2, 19].

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References

[1] U.Heinz, Phys. Rev. Lett. 51, 351 (1983); 56, 93(c) (1986); Ann. Phys.(N.Y.) 161,48 (1985); 168, 148 (1986);

H.-Th.Elze, M.Gyulassy and D.Vasak, Phys. Lett. B177, 402 (1986); Nucl. Phys. B276, 706 (1986); H.-Th. Elze, Z. Phys. C38, 211 (1988);

S.Mrówczyński, Phys. Rev. D39, 1940 (1989); J.-P.Blaizot and E.Iancu, Nucl. Phys. B390, 589 (1993); Phys. Rev. Lett. 72, 3317 (1994); Phys. Lett. B326, 138 (1994); D.F.Litim and C.Manuel, hep-ph/9906210.

[2] P.F.Kelly, Q.Liu,C C.Lucchesi and C.Manuel, Phys. Rev. Lett. 72, 3461 (1994); D50, 4209 (1994).

[3] D.J.Gross, R.D.Pisarski and L.G.Yaff, Rev. Mod. Phys. 53, 43 (1981); R.D.Pisarski, Phys. Rev. Lett. 63, 1129 (1989);
E.Braaten and R.D.Pisarski, Nucl. Phys. B337, 569 (1990); J.Frenkel and J.C.Taylor, Nucl. Phys. B334, 199 (1990).

[4] Zheng Xiaoping, Li Jiarong, Phys. Lett B409, 45 (1997); Nucl. Phys. A639, 705 (1998).

[5] J.-P.Blaizot and E.Iancu, Phys. Rev. Lett. 70, 3376 (1993).

[6] Zhang Xiaofei and Li Jiarong, Phys. Rev. C52, 964 (1995); J. Phys. G21, 1483 (1995).

[7] Yu.A.Markov and M.A.Markova, hep-ph/9902397, to published by Transp. Theor. Stat. Phys.

[8] V.N.Tsytovich, Theory of turbulent plasma, Plenum, New York, 1977.

[9] A.G.Sitenko, Fluctuations and Non-linear Wave interactions in plasma, Pergamon, Oxford, 1990.

[10] E.Braaten,R.Pisarski, Pyhs. Rev. Lett. 64, 1338 (1990).

[11] E.Braaten,R.Pisarski, Phys. Rev. D42, R2156 (1990).

[12] Zheng Xiaoping, Chen Jisheng and Li Jiarong, in preparation.

[13] H.-Th. Elze and U.Heinz, Phys. Rep. 183, 81 (1989).

[14] J.-P.Blaizot and E.Iancu, Nucl. Phys. B421, 565 (1994); 434, 662 (1995).

[15] D.F.Litim and C.Manuel, Phys. Rev. Lett. 82, 4991 (1999).

[16] Yu.A.Markov and M.A.Markova, hep-ph/9909505.

[17] D.Bödeker, Phys.Lett. B426, 351 (1998).

[18] S.Mrówczyński, Phys. Lett. B269, 383 (1991); Yuji Koike and T.Matsui, Phys. Rev. D45, 3237 (1992).

[19] R.D.Pisarski, hep-ph/9710371.